Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematics: analysis and approaches
Higher level
Paper 2

16 May 2025

Zone A morning | **Zone B** morning | **Zone C** morning

Candidate session number

2 hours

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all questions. Answers must be written within the answer boxes provided.
- Section B: answer all questions in the answer booklet provided. Fill in your session number on the front of the answer booklet, and attach it to this examination paper and your cover sheet using the tag provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

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Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions. Answers must be written within the answer boxes provided. Working may be continued below the lines, if necessary.

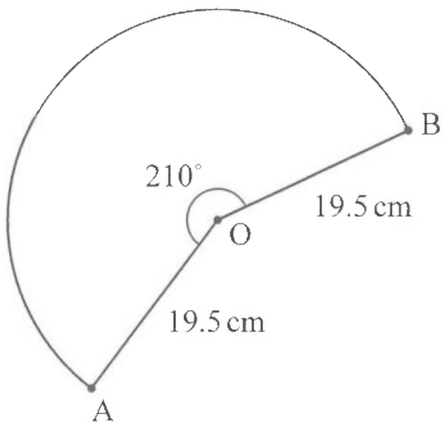
1. [Maximum mark: 6]

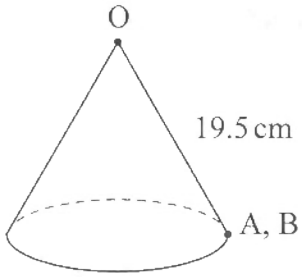
The points A and B lie on a circle, with centre O and radius $19.5\,\text{cm}$, such that $\text{B}\hat{\text{O}}\text{A} = 210^\circ$.

A piece of paper is cut into the shape of the sector BOA.

A hollow cone with no base is constructed from the sector by joining the points A and B. The sector forms the curved surface of the cone.

This is shown in the following diagrams.





Find

(a) the area of the sector BOA; [3]

(b) the radius of the cone. [3]

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(Question 1 continued)

2. [Maximum mark: 4]

Consider the function $f(x) = a\tan(2x) + b$, where $x \neq \frac{(2n+1)\pi}{4}$, $n \in \mathbb{Z}$ and $a, b \in \mathbb{R}$.

(a) Write down the period of f. [1]

The graph of $y = f(x)$ passes through the points $\left(\frac{\pi}{12}, 5\right)$ and $\left(\frac{\pi}{3}, 7\right)$.

(b) Find the value of a and the value of b. [3]

3. [Maximum mark: 6]

A population, P, has a rate of change modelled by $\frac{\mathrm{d}P}{\mathrm{d}t} = -104000\mathrm{e}^{-0.0145t}$, where t is the time measured in years since the **start** of 2022.

At the start of 2022, the population was 6.78×10^6.

Based on this model, find the predicted population at the start of 2026.

4. [Maximum mark: 8]

In a study, measurements for arm span, A cm, and foot length, F cm, are taken from a large group of adults.

For this group, the regression line of F on A is found to be $F = 0.335A - 32.6$, and the regression line of A on F is found to be $A = 2.89F + 99.3$. Each regression line passes through the mean point.

(a) By using an appropriate regression line, find an estimate of the arm span for an adult with a foot length of 19.8 cm. [2]

(b) For this group of adults, find the mean arm span and the mean foot length. [3]

The heights, H cm, of adults in the group can be modelled by a normal distribution with mean 163 cm and standard deviation σ cm.

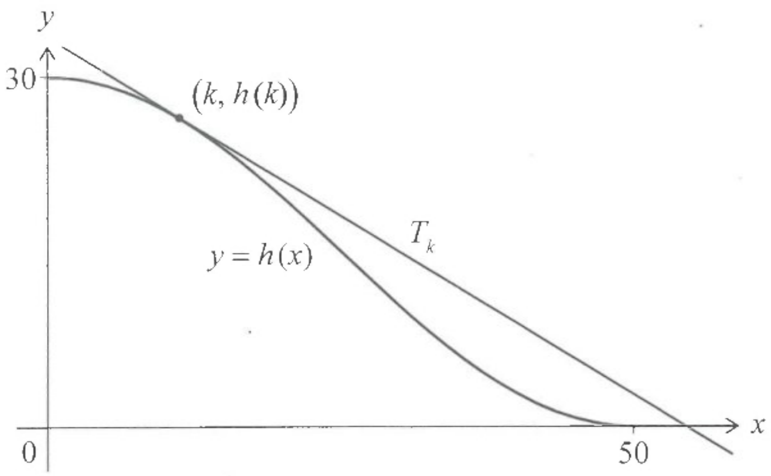
It is found that 88% of the group have a height between 153 cm and 173 cm.

(c) Find the value of σ. [3]

5. [Maximum mark: 6]

Consider the function $h(x) = 15\cos\left(\frac{\pi x}{50}\right) + 15$, where $0 \le x \le 50$.

The tangent, T_k, to the curve $y = h(x)$ at the point $(k, h(k))$ is shown on the following diagram.



(a) Find the gradient of T_k in terms of k. [3]

Consider the case where the angle between T_k and the x-axis is $\frac{\pi}{8}$ radians.

(b) Find the possible values of k. [3]

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6. [Maximum mark: 7]

Consider the function $f(x) = 4\cot x + \sin x$, where $0 < x < \pi$.

(a) (i) Write $f(x)$ in terms of $\sin x$ and $\cos x$.

(ii) Hence or otherwise, sketch the graph of $y = f(x)$, showing the value of the x-intercept. [3]

(b) Find the value of $f^{-1}(2)$. [1]

It is given that $\sec\alpha = 1.5$, where $0 < \alpha < \pi$.

(c) Find the value of $f(\alpha)$. [3]

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7. [Maximum mark: 5]

At 09:00 a helicopter is located at a point $(10, 3, 0.5)$ relative to a point O on horizontal ground. The x-direction is due east, the y-direction is due north and the z-direction is vertically upwards.

All distances are measured in kilometres.

The helicopter is flying at a constant height.

The helicopter's position relative to the point O is given by $\boldsymbol{r} = \begin{pmatrix} 10 \\ 3 \\ 0.5 \end{pmatrix} + 4t\begin{pmatrix} 10 \\ -25 \\ 0 \end{pmatrix}$, where t represents the time in hours since 09:00.

(a) Find the speed of the helicopter. [2]

At 10:00 the helicopter begins to descend.

During descent the helicopter's vertical height decreases at a constant rate of $16\,\text{km h}^{-1}$ and its horizontal velocity remains unchanged.

The angle of descent, β, is defined as the angle between the helicopter's direction of travel and the horizontal.

(b) Find β, giving your answer in degrees. [3]

8. [Maximum mark: 7]

Consider the functions f, g and h defined as follows for $t \in \mathbb{R}$.

$$f(t) = \sin(2t + 1)$$

$$g(t) = \sin(2t + 3)$$

$$h(t) = f(t) + g(t)$$

(a) Show that $h(t) = \operatorname{Im}(e^{2ti}(e^{i} + e^{3i}))$. [2]

(b) Write $e^{i} + e^{3i}$ in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. [2]

(c) Hence or otherwise, write $h(t)$ in the form $p\sin(2t + q)$, where $p > 0$ and $0 < q < 2\pi$. [3]

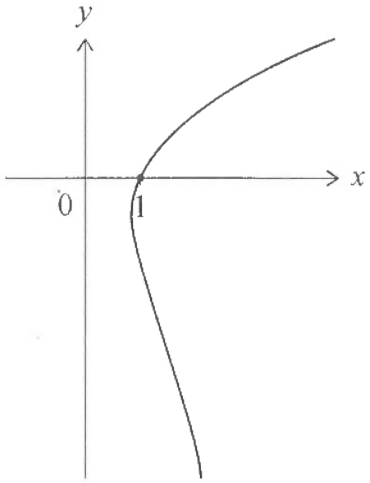
9. [Maximum mark: 6]

Consider the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x}{x^2 + y}$.

The solution curve passes through the point $(1, 0)$.

(a) Use Euler's method with a step value of 0.25 to estimate the value of y when $x = 2$. [3]

Part of the solution curve is shown in the following diagram.



(b) (i) Determine whether your answer to part (a) is an overestimate or an underestimate, justifying your answer.

(ii) Justify why the use of Euler's method starting at $(1, 0)$ does not lead to an estimate of the negative value of y when $x = 2$. [3]

Do **not** write solutions on this page.

Section B

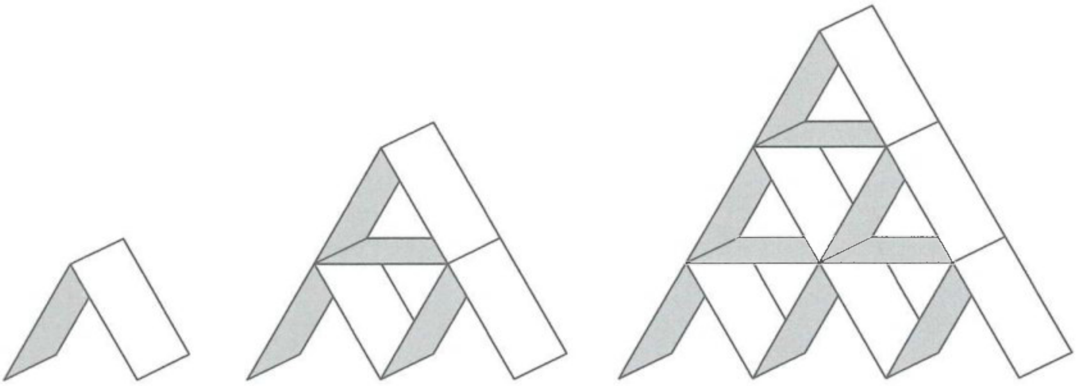
Answer **all** questions in the answer booklet provided. Please start each question on a new page.

10. [Maximum mark: 16]

Rectangular playing cards are stacked in the shape of a pyramid with n rows, where $n \geq 1$.

Some cards are placed horizontally and some cards are stacked at an angle of $60°$ to the horizontal.

The following diagrams represent pyramid stacks for $n = 1$, $n = 2$ and $n = 3$.



$n = 1$ $n = 2$ $n = 3$

Let t_n represent the number of cards used to create a pyramid stack with n rows.

(a) Write down t_3. [1]

(b) Find t_4. [2]

(c) Show that $t_n = \frac{n(3n+1)}{2}$. [3]

There are 52 cards in a full pack of playing cards.

(d) A complete pyramid stack is created using playing cards taken from 14 full packs. Find the maximum number of rows in this stack. [3]

(e) A complete pyramid stack is created using playing cards taken from full packs with no cards left over. Find the minimum number of rows in this stack. [2]

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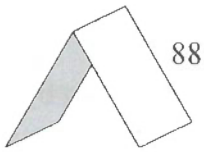
Do **not** write solutions on this page.

(Question 10 continued)

The long edge of each playing card measures $88\,\text{mm}$ as illustrated in the following diagram.



(f) Find the minimum number of cards needed to create a complete pyramid stack with a vertical height of more than 2 metres. The thickness of the cards may be ignored. [5]

Do **not** write solutions on this page.

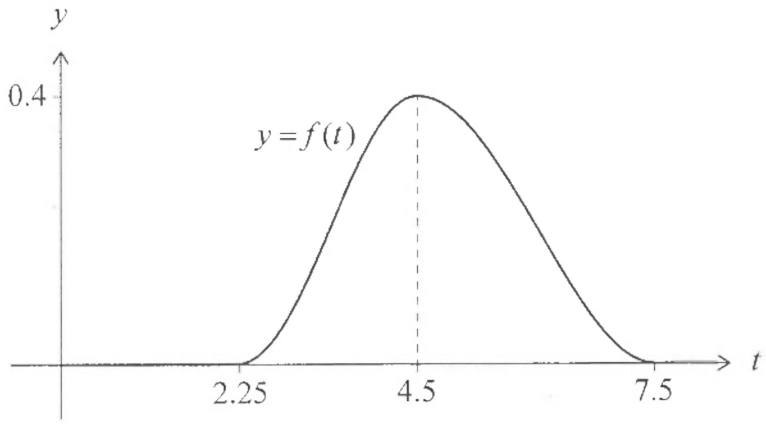
11. [Maximum mark: 19]

In a marathon race, the random variable T represents the time, in hours, taken for a runner to complete the race. No runner completes the race in less than 2.25 hours, and no runner completes it in more than 7.5 hours.

The probability distribution function for T is modelled by f, defined by

$$f(t)=\begin{cases}\frac{4}{21}\left(1-\cos\left(\frac{4\pi}{9}(t-2.25)\right)\right), & 2.25\le t<4.5\\ \frac{4}{21}\left(1+\cos\left(\frac{\pi}{3}(t-4.5)\right)\right), & 4.5\le t\le 7.5\\ 0, & \text{otherwise.}\end{cases}$$

The graph of f has a maximum point at $t=4.5$ as shown in the following diagram:



(a) (i) Find the value of $\int_{2.25}^{4.5} f(t)\,dt$.

(ii) Write down the mode of T.

(iii) Determine which is greater, the mode of T or the median of T, justifying your answer. [4]

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(Question 11 continued)

The runners who finish the race in 3.5 hours or less are considered to be fast runners.

(b) Find the probability that a runner chosen at random is a fast runner. [2]

(c) Find the probability that a fast runner chosen at random finishes the race in 3 hours or less. [3]

(d) Find the lower quartile of T. [3]

Each runner's time is converted to a score which is calculated as $a - bt$, where t represents their time in hours, and $a, b > 0$.

Consider the random variable P which represents the score of a runner. It is given that $\mathrm{E}(P) = 100$ and the maximum possible score is 150.

(e) Use $\mathrm{E}(T) = 4.723$ to determine the value of a and the value of b, giving your answers to the nearest integer. [5]

(f) Given also that $\mathrm{Var}(T) = 0.906$, find $\mathrm{Var}(P)$. [2]

Do **not** write solutions on this page.

12. [Maximum mark: 20]

Consider the family of functions f_n defined by $f_n(x) = \sum_{r=0}^{n} \left(-2x^2\right)^r$, where $x \in \mathbb{R}$ and $n \in \mathbb{N}$.

(a) Show that f_n is an even function for all values of n. [3]

(b) (i) Show that $f_3(x) = 1 - 2x^2 + 4x^4 - 8x^6$.

(ii) Write down a similar expression for $f_4(x)$ in ascending powers of x. [2]

Consider the function $f(x) = \lim_{n\to\infty} f_n(x)$ defined over the domain $-k < x < k$ where $k > 0$.

The largest possible value of k is K.

(c) (i) Find the value of K, giving your answer in exact form.

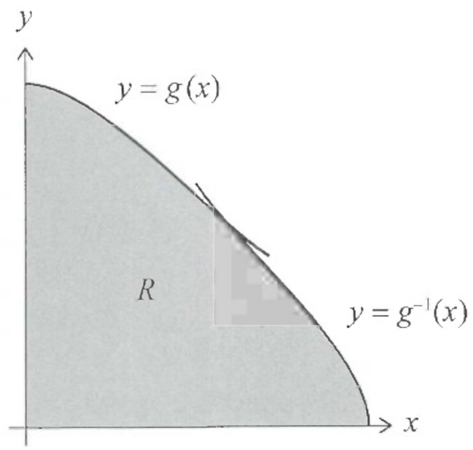
(ii) Express $f(x)$ as a rational function in the form $\frac{1}{a + bx^2}$, where a and b are constants to be determined. [5]

The function g is defined as $g(x) = f(x)$ for $0 \le x < K$.

(d) (i) Justify that g^{-1} exists.

(ii) Find $g^{-1}(x)$, giving its domain. [6]

The region R is completely enclosed by the curves $y = g(x)$, $y = g^{-1}(x)$ and the x- and y-axes, as shown on the following diagram.



(e) Find the area of R. [4]